TOPOLOGICAL FIELD THEORY

QUANTUM " "

TOY EXAMPLE: x, y $P(x) = \frac{1}{\sqrt{\pi}} e^{-x^2 - y^2}$

$$\langle x^n \rangle = \frac{1}{\sqrt{\pi}} \int x^n e^{-x^2} \qquad \begin{pmatrix} x \\ y \end{pmatrix} \to \underset{\underset{SO(2)}{\cap}}{M} \begin{pmatrix} x \\ y \end{pmatrix}$$

$\varphi(x)$, $\langle O \rangle$

---

① $\langle O_1(x_1) \dots O_n(x_n) \rangle_b = \int D\varphi(x)\, O_1(x_1) \dots e^{-S[\varphi]}$

$$S = \int dx\, h(\varphi, \partial\varphi)$$

$$b = (M, g, c_1, c_2, \dots)$$

② OPERATOR FORMALISM:

$$\int_{BC1}^{BC2} D\varphi \dots e^{-S[\varphi]} = \langle BC1 | T(\dots) | BC2 \rangle$$

$$\langle a | O | b \rangle = I(a, Ob)$$

$$\varphi(t_1) | BC1 \rangle = f(t_1) | BC1 \rangle$$

$$\varphi(t_1) = f(t_1)$$

$\delta | \varphi \rangle = \varepsilon S | \varphi \rangle$ "PASSIVE"

$\delta O = \varepsilon \{S, O\}$ "ACTIVE"

TOPOLOGICAL FT

$\langle O_1(x_1) \dots \rangle_b$ DOES NOT DEPEND ON g

GENERAL COORD. INV.

$$\delta x_i \Leftrightarrow \delta g$$

COHOMOLOGICAL FTs

SYMM. INVARIANT STATE : $S | \psi \rangle = 0$

" " VACUUM : $S | 0 \rangle = 0$

$$\langle 0 | O + \delta O | 0 \rangle = \langle 0 | O + \varepsilon [S, O] | 0 \rangle$$
$$= \langle 0 | O | 0 \rangle$$

1) $\exists$ SYMM. Q S.T. $Q^2 = 0$

2) PHYSICAL OPERATORS O_i: $\{Q, O_i\} = 0$

3) VACUUM IS Q-SYMMETRIC: $Q | 0 \rangle = 0$

$$\langle 0 | O_i \{Q, \Lambda\} O_j | 0 \rangle =$$
$$= \langle 0 | O_i Q \Lambda O_j | 0 \rangle \pm \dots$$
$$= \pm \langle 0 | Q O_i \Lambda O_j | 0 \rangle \pm \dots$$
$$= 0$$

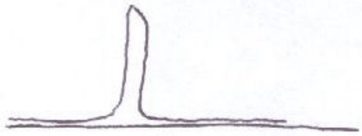
$$O_i \sim O_i + \{Q, \Lambda\}$$

4) $T_{\alpha\beta} = \frac{\delta S}{\delta h^{\alpha\beta}}$

$= \{Q, G_{\alpha\beta}\}$

$S = -\frac{1}{\hbar}\int L$

$L = \{Q, V\}$



$$\frac{\delta}{\delta h^{\alpha\beta}} \langle O_{i_1} \dots O_{i_n} \rangle = \frac{\delta}{\delta h^{\alpha\beta}} \int D\varphi \dots e^{-S[\varphi]}$$

$$= -\int D\varphi \dots T_{\alpha\beta}\, e^{-S[\varphi]}$$

$$= -\langle 0 | \dots \{Q, G_{\alpha\beta}\} | 0 \rangle$$

$$= 0$$

$$\frac{d}{d\hbar} \langle \dots \rangle = 0$$

CLASSICAL LIMIT IS EXACT

DESCENT EQNS.

$O_i(x)$ $\uparrow$

$$T_{\alpha\beta} = \{Q, G_{\alpha\beta}\}$$

$$P_\alpha = \int T_{\alpha 0}\, dx = \{Q, G_\alpha\}$$

$$\frac{d}{dx^\alpha} O_i = \{P_\alpha, O_i\}$$

$$O^{(0)} \to O^{(1)}_\alpha = \{G_\alpha, O^{(0)}\}$$

$$\frac{d}{dx^\alpha} O^{(0)} = [P_\alpha, O^{(0)}] = \{\{Q, G_\alpha\}, O^{(0)}\}$$

$$= \pm \{\{G_\alpha, O^{(0)}\}, Q\}$$

$$\pm \{\{O^{(0)}, Q\}, G_\alpha\}$$

$$= [Q, O^{(1)}_\alpha]$$

$$O^{(1)} = O^{(1)}_\alpha\, dx^\alpha$$

$$\Rightarrow dO^{(0)} = \{Q, O^{(1)}\}$$

$$0 = \{Q, \textstyle\int_\gamma O^{(1)}\}$$

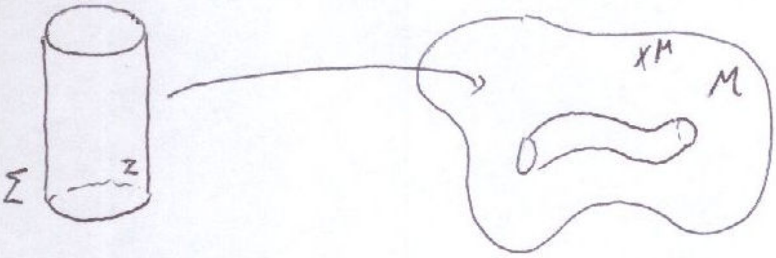
$\int_\gamma O^{(1)}$ PHYSICAL OPERATOR

$$\boxed{\begin{aligned} \{Q, O^{(0)}\} &= 0 \\ \{Q, O^{(1)}\} &= dO^{(0)} \\ \{Q, O^{(2)}\} &= dO^{(1)} \\ &\vdots \\ \{Q, O^{(d)}\} &= dO^{(n-1)} \\ 0 &= dO^{(n)} \end{aligned}}$$

$$\{Q, \int_M O_i^{(d)}\} = 0$$

$$L \to L + t^i O_i^{(d)}$$

STRING THEORY



WORLDSHEET

$X^i(z)$

$$S = \int d^2z \; g_{\mu\nu}(X) \, \eta^{\alpha\beta} \, \partial_\alpha X^\mu \partial_\beta X^\nu$$

$$+ \dots$$

WE WILL BE INTERESTED IN 2d SUSY QFT

$\Rightarrow$ COHOM. FT.

(2,2) - SUSY

$\Sigma = \mathbb{C}$ SUPER SPACE

$(z, \bar{z}, \theta^+, \theta^-, \bar{\theta}^+, \bar{\theta}^-)$ GRASSMANN:

$$\theta^+\theta^+ = 0$$

$$\theta^+\theta^- = -\theta^-\theta^+$$

$$\Phi(z, \bar{z}, \theta^\pm, \bar{\theta}^\pm) = \varphi(z,\bar{z}) + \psi_+(z,\bar{z})\,\theta^+ + \dots$$

ACTION: $S = \int d^2z \, d^4\theta \; K(\Phi^i, \bar{\Phi}^i)$

SYMMETRIES

$z = x^1 + i x^0$

$\downarrow$

"TIME"

1) SHIFTS $\quad \partial_z, \partial_{\bar{z}}$

$$H = -i(\partial_z - \partial_{\bar{z}})$$

$$P = -i(\partial_z + \partial_{\bar{z}})$$

2) ROTATIONS

$$M = 2z\partial_z - 2\bar{z}\partial_{\bar{z}} + \theta^+\frac{d}{d\theta^+} - \theta^-\frac{d}{d\theta^-} + \bar{\theta}^+\frac{d}{d\bar{\theta}^+} - \bar{\theta}^-\frac{d}{d\bar{\theta}^-}$$

$$[M, H] = -2P$$

$$[M, P] = -2H$$

3) FERMIONIC SHIFTS / ROTATIONS

$$Q_\pm = \frac{\partial}{\partial\theta^\pm} + i\bar{\theta}^\pm\partial_\pm$$

$$\bar{Q}_\pm = -\frac{\partial}{\partial\bar{\theta}^\pm} - i\theta^\pm\partial_\pm$$

$$\bar{\theta}^+\theta^+$$

$$D_\pm = \frac{\partial}{\partial\theta^\pm} - i\bar{\theta}^\pm\partial_\pm$$

$$\bar{D}_\pm = -\frac{\partial}{\partial\bar{\theta}^\pm} + i\theta^\pm\partial_\pm$$

CHIRAL FIELDS:

$\bar{D}_\pm \Phi = 0$ 16 DOF → 4 DOF

$$\Phi(z', \bar{z}') = \varphi(z', \bar{z}') + \psi_+(z', \bar{z}')\theta^+ + \psi_-(z', \bar{z}')\theta^- + F\theta^+\theta^-$$

$$z' = z - i\theta^+\bar{\theta}^+$$

$$\bar{z}' = \bar{z} - i\theta^-\bar{\theta}^-$$

$\{Q, D\} = 0$ $N = (2,2)$ SUSY

$$\bar{D}_\pm (Q_\pm \Phi) = 0$$

$$S = \int d^2 z \, d^4\theta \, K(\Phi^i, \bar{\Phi}^i)$$

↳ CHIRAL